Guessing that the Riemann Hypothesis is unprovable

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Abstract

Riemann Hypothesis has been the unsolved conjecture for 164 years. This conjecture is the last one of conjectures without proof in "Ueber die Anzahl der Primzahlen unter einer gegebenen Grösse"(B. Riemann). The statement is the real part of the non-trivial zero points of the Riemann Zeta function is 1/2. Very famous and difficult this conjecture has not been solved by many mathematicians for many years. In this paper, I guess the independence (unprovability) of the Riemann Hypothesis.

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I define Möbius function $\mu(n)$ as

$$\mu(n) := \begin{cases} 1 & product\ of\ even\ primes \\ -1 & product\ of\ odd\ primes \\ 0 & divisible\ by\ the\ square\ of\ a\ prime\ number \end{cases}$$

The Riemann Hypothesis is that the real part of the nontrivial zero point of the ζ function is 1/2 (Ivić[1] p44).

Theorem 1. *(Ivić[1]p48,Titchmarsh[4] p370, Theorem 14.25)*

$$the\ Riemann\ Hypothesis \Leftrightarrow \sum_{n=1}^{m} \mu(n) = O(m^{\frac{1}{2}+\epsilon})$$

Proposition 1.

$$\sum_{n=1}^{m} \mu(n) = O(m^{\frac{1}{2}+\epsilon})$$

I guess this proposition's unprovability. This proposition is equivalent to the Riemann Hypothesis (by theorem 1).

I define "sum of the distorted Möbius function" as $\sum_{n\leq N-1}\mu(n)+f(N)$ that is in the summation of Möbius function (up io N) add (or subtract) $f(N)$ instead of $\mu(N)(\mu(N)\neq 0)$. I take $N\to\infty(\mu(N)\neq 0).f(N)$ and $\mu(N)$ are taken as same sign. The value at $N*M$ is taken as $f(N)*\mu(M)$. N to ∞, this operation can be repeated. Only needed value is at N. The sum of Möbius functions at $N\to\infty(\mu(N)\neq 0)$ is set arbitrary to some extent. (I use two cases afterwards, primary case is $|\sum_{n\leq N-1}\mu(n)+f(N)|<KN^{\frac{1}{2}+\epsilon}$, $N\to\infty(\mu(N)\neq 0)$ case. $f(N)$ is definded as $|\sum_{n\leq N-1}\mu(n)+f(N)|<KN^{\frac{1}{2}+\epsilon}$, sercondary case is $|\sum_{n\leq N-1}\mu(n)+f(N)|=KN^{\frac{2}{3}}$, $N\to\infty(\mu(N)\neq 0)$ case. $f(N)$ is definded as $|\sum_{n\leq N-1}\mu(n)+f(N)|=KN^{\frac{2}{3}}$. The case $|\sum_{n\leq N-1}\mu(n)+f(N)|>N$ is actually impossible value. I do not think this case.)

conjecture 1. *The Riemann Hypothesis is impossible to prove. In other words, the Riemann Hypothesis is an "independent proposition."*

Consider Riemann Hypothesis of Möbius function case (proposition 1). Let ZFC (K.Kunen[2] introduction §1) be the entire axiomatic system.

First, the Riemann Hypothesis can be disproved case, this is one possibility. Later, for finite range, there are not counter examples (to proposition 1).

I think next two models.

Model A is the case

$$\sum_{n=1}^{m}\mu(n)=O(m^{\frac{1}{2}+\epsilon})$$

holds for all m (containing $m=\infty$).

Model B is the case

$$\sum_{n=1}^{m}\mu(n)=O(m^{\frac{1}{2}+\epsilon})$$

does not hold at $m=\infty$.

There are no counter examples for proposition 1 for finite range, so Model A holds for all finite m. I think $m=N$ case, I take "sum of the distorted Möbius function" to satiszfy $|\sum_{n\leq N-1}\mu(n)+f(N)|<KN^{\frac{1}{2}+\epsilon}$, $N\to\infty(\mu(N)\neq 0)$. So Model A holds at infinity. Model A is consistent in

ZFC. (Model A consists of non contradicting formulas. So that is consistent. (K.Kunen[2], Introduction §1))

I think $m = N$ case, I take "sum of the distorted Möbius function" to satisfy $|\sum_{n \leq N-1} \mu(n) + f(N)| = KN^{\frac{2}{3}}$. $KN^{\frac{2}{3}} = O(N^{\frac{1}{2}+\epsilon})$ does not hold, so $N \to \infty(\mu(N) \neq 0)$, Model B holds at infinity. Model B is consistent in ZFC.

Like this condition, the Riemann Hypothesis is unprovable and is independent. (K.Kunen[2], Introduction §1)

There are two possibilities the Riemann Hypothesis can be disproved or cannot be proven.

Special thanks: I was very grateful to my friend H. Tokitu for translating in English. I would like to express my gratitude to him.

References

[1] "The Riemann Hypothesis", Ivić. A, Dover Press

[2] "Set Theory An Introduction To Independence Proofs", Kunen. K. Studies in Logic and the Foundations of Mathematics,Volume 102.

[3] 'Ueber die Anzahl der Primzahlen unter einer gegebenen Grösse", Riemann Bernhard, Monatsberichte der Berliner Akademie. In Gesammelte Werke, Teubner, Leipzig (1892)

[4] "The Theory of the Riemann Zeta function", Titchmarsh, E, C,, Clarendon Press Oxford (1996)