Half proof of Collatz problem

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Abstract

We prove Collatz sequence has not general m-cycle. Already proved result is that there is no less than 68-cycle. We can not prove the possibility Collatz sequence goes to infinity.

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In this paper, we prove partial result of Collatz problem. This problem is bred by Lothar Collatz at 1937. For 80 years, proved best result is 68-cycle case. We prove general m-cycle case.

Collatz problem

$$\begin{cases} n \textit{ is even number} & \Rightarrow \textit{divides } 2 \\ n \textit{ is odd number} & \Rightarrow \textit{times } 3 \textit{ plus } 1 \end{cases}$$

repeat this prosess, this sequence reaches to 1

Next result is known already.

Theorem 1.1. *(68-cycle)*
Except trivial case, some number does not go to same number in the case less than 68 times increase from odd number less than 68 times decrease from even number.

example:

$$43 \to 130 \to 65 \to 196 \to 49 \to 148 \to 37 \to 112 \to 7 \to 22 \to 11$$
$$\to 34 \to 17 \to 52 \to 13 \to$$

This sequence has possibility to be the part of 8 or more cycle.

Theorem 1.2. *(Half proof of Collatz problem)*
Except trivial case, In the Collatz sequence, some number does not go to same number

Corollary 1.1. *Collatz sequence is gradually increase and goes to infinity or gradually decrease and goes to 1.*

proof. We prove the theorem. We assume next formula.

$$(\frac{1}{2})^n(\frac{3}{2})^m N + \alpha = M$$

$$\alpha = (\frac{1}{2})^{n_1} + (\frac{1}{2})^{n_2}(\frac{3}{2}) + (\frac{1}{2})^{n_3}(\frac{3}{2})^2 + \cdots + (\frac{1}{2})^{n_{m-1}}(\frac{3}{2})^{m-1}$$
$$n_1 \leq n_2 \leq n_3 \leq \cdots \leq n_{m-1}$$

M is the number the $(m-1)$th odd number from N.

example.
$N = 43, M = 13$ case.

$$M(=13) = (\frac{1}{2})^6(\frac{3}{2})^7 N(=43) + \alpha$$

$$\alpha = (\frac{1}{2})^2 + (\frac{1}{2})^2(\frac{3}{2}) + (\frac{1}{2})^2(\frac{3}{2})^2 + (\frac{1}{2})^5(\frac{3}{2})^3 + (\frac{1}{2})^6(\frac{3}{2})^4$$
$$+(\frac{1}{2})^7(\frac{3}{2})^5 + (\frac{1}{2})^7(\frac{3}{2})^6$$

In this formula, $\frac{1}{2}$ appear at the time $\frac{3N+1}{2}$ occur. Thereafter, we multiply $\frac{3}{2}, \frac{1}{2}$ according to calculation. This formula easily can be checked by computer.

We assume that M and N is equal and lead contradiction. For easy to understand, we take $M = 13, N = 43$ and assume $M = N$. Ofcourse,$43 \neq 13$.

$$N = (\frac{1}{2})^6(\frac{3}{2})^7 N + \alpha$$

We multiply 2^{13}.

$$2^{13}N = 3^7 N + 2^{11} + 2^{10}3 + 2^9 3^2 + 2^5 3^3 + 2^3 3^4 + 23^5 + 3^6$$

We calculate α's value. $\alpha = 1.520386\cdots$. $N = M$,

$$N(1 - (\frac{1}{2})^n 3^m) = \alpha$$

$$N(2^n - 3^m) = 2^{n-l} + 2^{n-l'}3 + \cdots + 3^{m-1}$$

We assume $m \geq 2$.The case $N = 1, n = 2, m = 1, l = 2$ exists.

$$2^{13}N - 3^7 N = 2^{11} + 2^{10}3 + 2^9 3^2 + 2^5 3^3 + 2^3 3^4 + 23^5 + 3^6$$

We take $2^{14} = 16384$, $16384 + N = 16384 + 43 = 16427$. This 16427 moves just like 43.The value α is common. If 43 is roop, then 16427 is roop, too.

$$2^{11} + 2^{10}3 + 2^9 3^2 + 2^5 3^3 + 2^3 3^4 + 23^5 + 3^6$$

is multiple of 43. And it is multiple of 16427, too. This is the contradiction.This case can not exist. For all natural number n, it can not exist. $m-$cycle can not exist. □

Finally we calculate 1-cycle.

$$N = (\frac{1}{2})^n \frac{3}{2} N + (\frac{1}{2})^{n+1}$$

multiply 2^{n+1}.

$$2^{n+1}N = 3N + 1 \Rightarrow ((2^{n+1}) - 3)N = 1$$

$$(2^{n+1} - 3)N = 1, n = 1, N = 1$$

1 is trivial sequence $1 \to 4 \to 1 \to 4$. This result is easilly checked.

$$1 = (\frac{1}{2})\frac{3}{2} \times 1 + (\frac{1}{2})^2 = \frac{3}{4} + \frac{1}{4} = 1$$