

The relation of primitive root and cyclic decimal

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July 24, 2016

Abstract

We prove one conjecture. This paper consists 2 example and prove the conjecture.

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We begin

$$\frac{1}{7} = 0.14285714857 \dots$$

$10 = 3 + 7$ is divided by 7 as 1 and the rest is 3. 30 is divided by 7 as 4 and the rest is 2. 20 is divided by 7 as 2 and the rest is 6. This is correspond to $3 \rightarrow 2 \rightarrow 6$. So, that 3 is the primitive root of 7 equals to recurring decimal of $\frac{1}{7}$. General case is not so easy

$17 + 7 = 24$ case, we calculate 24 numeration. 24 is divided by 17 as 1 and the rest is 7. $7 \times 24 = 168$ is divided by 7 as 9 and the rest is 15.

$$7 \rightarrow 15 \rightarrow 3 \rightarrow 4 \rightarrow 11 \rightarrow 9 \rightarrow 12 \rightarrow 16 \rightarrow 10 \rightarrow 2 \rightarrow 14 \rightarrow 13 \rightarrow 6 \\ \rightarrow 8 \rightarrow 5 \rightarrow 1 \dots$$

In this case, 7 is primitive root of 17. So recurring decimal in 24 numeration is repeat 16.

conjecture

p is prime and more than 5. 10 is primitive root of p is equal to $\frac{1}{p}$'s repeating decimal length is $p - 1$

proof. We see next case.

$$\frac{1}{17} = 0.0588235294117647 \dots$$

$p = 17$ case, 10 is the primitive root. First, $10 - 17 = -7$. Usually, in this case digit is increase 1. $100/17 = 5$ and the rest is $15 \equiv (-7)^2 = 49 \pmod{17}$. $150/17 = 8$ and the rest $14 \equiv (-7)^3 = -343 \pmod{17}$ We got the $p - 1 = 16$ cycle.

□